

USING HERMITE COLLOCATION TO SOLVE CONVECTION-DIFFUSION PROBLEMS

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Convection-diffusion equations occur in a wide variety of science and engineering applications. When convection is the dominant process, the accurate numerical solution of such equations is a difficult task. Physically absurd oscillations are often present in such solutions. These may be ameliorated by employing “upstream weighting” on the convective term, a technique which often introduces the unwanted effect of “numerical diffusion,” which “smears” the sharp solution profile that one hopes the numerical solution will capture.

In this presentation, we use Hermite collocation to discretize our differential equations and employ the upstream weighting technique. For constant coefficient problems on a uniform mesh, we derive analytical formulas for the systems of linear algebraic equations that arise from this discretization. We discuss how to select the upstreaming parameter in an optimal way, the result of which is that the numerical solution both accurately captures the sharp solution profile and avoids unwanted oscillations. For the case of non-homogeneous equations, we discuss the issue of sampling the forcing function in an optimal manner. We then extend these results to the setting where the coefficients are non-constant. The result of our explorations is that we obtain highly accurate solutions that suffer from neither numerical diffusion or oscillations, particularly when convection dominates.